QP CODE: 22001452

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Reg No : .....

Name : .....

M Sc DEGREE (CSS) EXAMINATION, JULY 2022

First Semester

CORE - ME010105 - GRAPH THEORY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

2CCC8E65

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define (a) complete graph(b) trivial graph (c) bipartite graph (d) clique of a graph (e) complement of a graph
2. Let H be a spanning subgraph of G . Does it imply that L(H) is a spanning subgraph of L(G). Justiy your answer.
3. a. Define cut vertex and cut edge of a graph.

   b. Show that a graph has a cut vertex need not imply it has a cut edge.
4. (a) Define (i) vertex connectivity of a graph,

   (ii) r-connected graph

   (b) Given examples of cubic graphs G_1, G_2, G_3 with $\kappa(G_1) = 1, \kappa(G_2) = 2, \kappa(G_3) = 3$
5. (a) Define edge conraction in graphs and write the formula for finding the number of spanning trees in a connected labelled graph.

   (b) Find $\tau(C_4)$
6. Does there exists an Eulerian graph with (i) an even number of vertices and an odd number of edges and (ii) an odd number of vertices and an even number of edges. Draw such a graph if it exists.
7. Show by an example that if closure of a graph G is Hamiltonian, then G is Hamiltonian.
8. Define chromatic number of a graph G. Explain with an example.
9. *What you mean by the faces of a planar graph.*
10. Write the adjacency matrix of K_5

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let G be a graph. Define distance function d on $V(G)$ and show that d is metric on $V(G)$

12. Find the order and size of $G_1 \square G_2$

13. Define cyclical edge connectivity of a graph and show that cyclical edge connectivity of the Peterson graph is five.

14. Find a minimal spanning tree of G whose weight matrix is given by

$$\begin{bmatrix} \infty & 385 & 425 & 1035 & 708 & 644 \\ 385 & \infty & 255 & 740 & 773 & 329 \\ 425 & 255 & \infty & 679 & 531 & \infty \\ 1035 & 740 & 679 & \infty & 816 & 860 \\ 708 & 773 & 531 & 816 & \infty & 1095 \\ 644 & 329 & \infty & 860 & 1095 & \infty \end{bmatrix}$$

using Prim's algorithm

15. Explain Hamilton's "Around the World" Game.

16. For any simple graph G, prove that $2\sqrt{n} \leq \chi + \chi^c \leq n+1$ and $n \leq \chi\chi^c \leq ((n+1)/2)^2$

17. *State and prove Euler formula for a connected plane graph.*

18. *Show that $K_{3,3}$ is a nonplanar graph using the Jordan Curve Theorem.*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19.

a. State and prove Moon's theorem.

b. State and prove Redei's theorem.

20.

a. Show that every tree has a centre consisting of either a single vertex or two adjacent vertices.

b. Show that every connected graph contains a spanning tree.

c. Prove that for a connected graph G , r(G) ≤ diam(G) ≤ 2r(G).

21. State and prove Brook's theorem.

22. *State and prove Heawood's five color theorem.*

(2×5=10 weightage)